

**PROBABILITY, UNCERTAINTY AND  
QUANTITATIVE RISK**

June 22-25, 2015, SDU, Ji'nan & Weihai, China

Website: <http://rida.sdu.edu.cn/PUQR2015/>

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概率, 不确定性和数量风险国际会议  
**Probability, Uncertainty and Quantitative Risk**

June 22-25, 2015, SDU, Ji'nan & Weihai, China

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Rainer Buckdahn (UBO and SDU)  
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 Shandong University, Weihai

**Invited Speakers:**

Rainer Buckdahn	University of Brest, France
Xia Chen	Jilin University, China & University of Tennessee, USA
Zengjing Chen	Shandong University, China
Zhenqing Chen	University of Washington, USA
Samuel Drapeau	Shanghai Jiao Tong University, China
Ernst Eberlein	University of Freiburg, Germany
Emmanuel Gobet	Ecole Polytechnique, France
Jianhui Huang	The Hong Kong Polytechnic University, China
Shaolin Ji	Shandong University, China
Yury Kutoyants	University of Maine, France
Zhengyan Lin	Zhejiang University, China
Qi Lv	Sichuan University, China
Jin Ma	University of Southern California, USA
Shige Peng	Shandong University, China
Huy�en Pham	Paris Diderot University, France

Shanjian Tang	Fudan University, China
Ciprian Tudor	Université Lille 1, France
Zhen Wu	Shandong University, China
Jie Xiong	University of Macau, China
Zuoquan Xu	The Hong Kong Polytechnic University, China
Jiongmin Yong	University of Central Florida, USA
Jianfeng Zhang	University of Southern California, USA
Xu Zhang	Sichuan University, China
Huaizhong Zhao	Loughborough University, UK

Probability, Uncertainty and Quantitative Risk-Schedule							
MONDAY June 22		TUESDAY June 23		WEDNESDAY June 24		THURSDAY June 25	
08:30-09:00	Registration	09:00-09:40	Z.Q.Chen	09:00-09:40	J.Ma	09:00-09:40	X.Zhang
09:00-09:30	Opening: S.G.Peng						
09:30-09:40	Take Photo						
09:40-10:20	H.Pham	09:40-10:20	J.M.Yong	09:40-10:20	S.J.Tang	09:40-10:20	Z.J.Chen
10:20-10:50	<i>Coffee Break</i>	10:20-10:50	<i>Coffee Break</i>	10:20-10:50	<i>Coffee Break</i>	10:20-10:50	<i>Coffee Break</i>
10:50-11:30	E.Eberlein	10:50-11:30	E.Gobet	10:50-11:30	J.F.Zhang	10:50-11:30	Q.Lv
11:30-12:10	H.Z.Zhao	11:30-12:10	Z.Wu	11:30-12:10	S.Drapeau	11:30-12:10	R.Buckdahn
12:10-14:00	<i>Lunch Break</i>	12:10-14:00	<i>Lunch Break</i>	12:10-14:00	<i>Lunch Break</i>	12:10-14:00	<i>Lunch Break</i>
14:00-14:40	Y.Kutoyants	14:00-14:40	C.Tudor	14:00-14:40	Z.Y.Lin	Departure	
14:40-15:20	X.Chen	14:40-15:20	J.Xiong	14:40-15:20	S.L.Ji		
15:20-15:40	<i>Coffee Break</i>	15:20-15:40	<i>Coffee Break</i>	15:20-15:50	<i>Coffee Break</i>		
15:40-17:40	Journal	15:40-17:40	Journal	15:50-16:30	Z.Q.Xu		
	Discussion		Discussion	16:30-17:10	J.H.Huang		

\* Conference Banquet: 18:30-20:30, June 22. Lunch: 12:10-14:00. Dinner: 18:30-20:30. Lunch & Dinner: Buffet at International Academic Center.

## Schedule

**Attention:** All the talks will be given in International Academic Center, No. 47 in the map in the Appendix.

**June 22<sup>nd</sup>**

### Morning

9:00-9:30 Opening: Shige Peng

9:30-9:40 Take photo

9:40-10:20

Speaker: Huy en Pham, Paris Diderot University, France

Title: BSDEs with diffusion constraint and viscous Hamilton-Jacobi equations with unbounded data

10:20-10:50 Tea & Coffee Break

10:50-11:30

Speaker: Ernst Eberlein, University of Freiburg, Germany

Title: Two price valuation in continuous time

11:30-12:10

Speaker: Huaizhong Zhao, Loughborough University, UK

Title: Random periodic processes, periodic measures and ergodicity

12:10-14:00 Lunch

### Afternoon

14:00-14:40

Speaker: Yury Kutoyants, University of Maine, France

Title: On multi-step MLE-processes in the problem of estimation of the solution of BSDE

14:40-15:20

Speaker: Xia Chen, Jilin University, China & University of Tennessee, USA

Title: Exponential asymptotics for time-space Hamiltonians

15:20-15:40 Tea & Coffee Break

15:40-17:40 Journal Discussion

18:30-20:30 Conference Banquet

**June 23<sup>rd</sup>**

### Morning

9:00-9:40

Speaker: Zhenqing Chen, University of Washington, USA

Title: Fractional time stochastic partial differential equations

9:40-10:20

Speaker: Jiongmin Yong, University of Central Florida, USA

Title: Comparison theorems for backward stochastic Volterra integral equations and dynamic risk measures

10:20-10:50 Tea & Coffee Break

10:50-11:30

Speaker: Emmanuel Gobet, Ecole Polytechnique, France

Title: Rare event simulation related to financial risks: efficient estimation and sensitivity analysis

11:30-12:10

Speaker: Zhen Wu, Shandong University, China

Title: Partially observable optimal control problem of forward-backward stochastic control systems

12:10-14:00 Lunch

## **Afternoon**

14:00-14:40

Speaker: Ciprian Tudor, Université Lille 1, France

Title: On the law and sample paths of the solution to the heat equation with fractional noise

14:40-15:20

Speaker: Jie Xiong, University of Macau, China

Title: Large deviation and moderate deviation principle for some measure-valued processes

15:20-15:40 Tea & Coffee Break

15:40-17:40 Journal Discussion

18:30-20:30 Dinner

## **June 24<sup>th</sup>**

### **Morning**

9:00-9:40

Speaker: Jin Ma, University of Southern California, USA

Title: Conditional mean-field SDEs and related stochastic control problems

9:40-10:20

Speaker: Shanjian Tang, Fudan University, China

Title: Multi-dimensional backward stochastic differential equations of diagonally quadratic generators

10:20-10:50 Tea & Coffee Break

10:50-11:30

Speaker: Jianfeng Zhang, University of Southern California, USA

Title: Some thoughts about pathwise stochastic analysis

11:30-12:10

Speaker: Samuel Drapeau, Shanghai Jiao Tong University, China

Title: Multivariate shortfall risk and monetary risk allocation

12:10-14:00 Lunch

### **Afternoon**

14:00-14:40

Speaker: Zhengyan Lin, Zhejiang University, China

Title: On weak convergence of stochastic processes to stochastic integrals

14:40-15:20

Speaker: Shaolin Ji, Shandong University, China

Title: Dynamic programming principle for stochastic recursive optimal control problem under G-framework

15:20-15:50 Tea & Coffee Break

15:50-16:30

Speaker: Zuoquan Xu, The Hong Kong Polytechnic University, China

Title: Rank-dependent utility maximization with bounded risk exposure

16:30-17:10

Speaker: Jianhui Huang, The Hong Kong Polytechnic University, China

Title: Backward mean-field games with major and minor agents

18:30-20:30 Dinner

### **June 25<sup>th</sup>**

#### **Morning**

9:00-9:40

Speaker: Xu Zhang, Sichuan University, China

Title: First and second order necessary conditions for stochastic optimal controls

9:40-10:20



Speaker: Zengjing Chen, Shandong University, China

Title: The laws of large numbers without additivity: necessary and sufficient conditions

10:20-10:50 Tea & Coffee Break

10:50-11:30

Speaker: Qi Lv, Sichuan University, China

Title: Observability estimate of backward stochastic schrödinger equations

11:30-12:10

Speaker: Rainer Buckdahn, Université de Bretagne Occidentale, France

Title: Peng's stochastic maximum principle for mean-field stochastic control problems

12:10-14:00 Lunch

### **Afternoon**

Departure

## Abstracts of Talks

### Peng's stochastic maximum principle for mean-field stochastic control problems

Rainer Buckdahn

Laboratoire de Mathématiques, Université de Bretagne Occidentale, France  
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**Abstract:** The talk extends Peng's stochastic maximum principle from classical stochastic control problems to those in which the coefficients of the dynamics of the controlled state process do not only depend on the state process and the control themselves but also on the law of the control state process. The characterization of the optimal control, which is obtained, extends the corresponding result by Shige Peng and contains also extra-terms coming from the mean-field character of the stochastic control problem.

(Joint work with Juan Li (SDU, Weihai), Jin Ma (USC, Los Angeles).)

### Exponential asymptotics for time-space Hamiltonians

Xia Chen

Jilin University, China & University of Tennessee, USA  
*xchen@math.utk.edu*

**Abstract:** In this talk, we investigate the long time asymptotics of the exponential moment for the time-space Hamiltonians appearing as the covariance functions of the fractional Gaussian noises. Our study is motivated partially by the intermittency problem for the parabolic Anderson models with the fractional Gaussian potentials.

(Joint work with Yaozhong Hu, Jian Song and Fei Xing.)

## **The laws of large numbers without additivity: necessary and sufficient conditions**

Zengjing Chen

School of Mathematics, Shandong University, China

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**Abstract:** In this paper, we investigate strong laws of large numbers for capacities under weakened conditions. We obtain two sufficient and nearly necessary conditions for strong laws of large numbers. One is a sufficient and nearly necessary condition under which any cluster point of empirical average lies, with probability (capacity) one, between upper and lower Choquet expectations; The other is a sufficient and nearly necessary under which the interval between upper and lower Choquet expectations is the unique smallest interval in which any cluster point of empirical average lies with probability (capacity) one. Furthermore, we study some examples to explain the strong laws of large numbers for capacities.

(Joint work with Gaofeng Zong.)

## **Fractional time stochastic partial differential equations**

Zhenqing Chen

University of Washington, USA

*zqchen@uw.edu*

**Abstract:** In this talk, I will introduce a class of stochastic partial differential equations (SPDEs) with fractional time-derivatives, and present the  $L_2$ -theory of the equations. This class of SPDEs can be used to describe random effects on transport of particles in medium with thermal memory or particles subject to sticking and trapping.

(This is based on joint work with K. H. Kim and P. Kim.)

## **Multivariate shortfall risk and monetary risk allocation**

Samuel Drapeau

Shanghai Advanced Institute of Finance, Shanghai Jiao Tong University, China  
*sdrapeau@saif.sjtu.edu.cn*

**Abstract:** The financial 2007/2008 crisis revealed that too few attention was paid to a sound risk and uncertainty management in particular in its systemic dimension. In this work, we present a risk measure designed to address the global and intrinsic risk of multidimensional interconnected system such as banks or counter-party risk in a central clearing house. The goal is two fold: on the one hand, it provides the total amount of liquidity that has to be reserved for the system to overcome financial stress situations. On the other hand, it addresses the respective amount that each member has to reserve in function of their exposure to the whole system and the systemic risk they put on the system. We finally address the quantitative aspects by presenting how these high dimensional computations can be solved in an efficient manner using Fourier methods.

(Joint work with Y. Armenti, S. Crepey and A. Papapantoleon.)

## **Two price valuation in continuous time**

Ernst Eberlein

University of Freiburg, Germany  
*eberlein@stochastik.uni-freiburg.de*

**Abstract:** In the classical valuation theory the law of one price prevails and market participants trade freely in both directions at the same price. This approach is appropriate for highly liquid markets. In the absence of perfect liquidity the law of one price should be replaced by a two price valuation theory where market participants continue to trade freely with the market but the terms of trade now depend on the direction of the trade. We develop here a static as well as a continuous time theory for two price economies. The two prices are termed bid and ask or lower and upper price but they should not be confused with the literature relating bid-ask spreads to transaction costs or other frictions involved in modeling financial markets. The bid price arises as the infimum of test valuations whereas the ask price is the supremum of such valuations. The two prices are related to nonlinear expectations. Probability as well as measure distortions are used to make this approach operational. We consider specific models where the uncertainty is given by purely discontinuous Lévy processes. The approach is illustrated to price stochastic perpetuities, i.e. contracts with no apparent maturity, and to value compound Poisson processes of insurance loss liabilities.

(Joint work with Dilip Madan, Martijn Pistorius, Wim Schoutens and Marc Yor.)

## **Rare event simulation related to financial risks: efficient estimation and sensitivity analysis**

Emmanuel Gobet

Applied Mathematics, Ecole Polytechnique, France

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**Abstract:** We develop the reversible shaking transformation methods of Gobet and Liu (2014) to estimate the rare event probability arising in different financial risk settings driven by general Gaussian noise. The underlying Markov chains introduced in our approaches take values directly in the path space. We provide theoretical justification for few key properties of these Markov chains which are required for their ergodicity. Further, using these properties, we prove consistency results for the simulation estimator. The examples in our work cover usual semi-martingale stochastic models (not necessarily Markovian) driven by Brownian motion, and, also fractional Brownian motion based models to address various financial risks. Our approach also handles the important problem of sensitivities of rare event probability. We compare our numerical studies to the already existing results and demonstrate improved computational performance.

(Joint work with A. Agarwal, S. De Marco and G. Liu.)

## **Backward mean-field games with major and minor agents**

Jianhui Huang

Department of Applied Mathematics, The Hong Kong Polytechnic University, HK, China

*majhuang@polyu.edu.hk*

**Abstract:** This paper investigates the backward-forward linear-quadratic-Gaussian (LQG) games with major and minor players. The dynamics of major player is given by a backward stochastic differential equation (BSDE); while dynamics of minor players are described by (forward) stochastic differential equations (SDEs). A backward-forward stochastic differential equation (BFSDE) system is established in which a large number of negligible agents are coupled in

their dynamics via the state average. The problem when major player takes into account the relative performance by comparison to minor players is under consideration. Some auxiliary mean-field forward-backward stochastic differential equation (FBSDE) system are considered and analyzed instead of involving the fixed-point analysis. The decentralized strategies are derived, which are also shown to satisfy the  $\epsilon$ -Nash equilibrium property.

(Joint work with Shujun Wang, and Zhen Wu.)

## **Dynamic programming principle for stochastic recursive optimal control problem under G-framework**

Shaolin Ji

School of Mathematics, Shandong University, China

*jsl@sdu.edu.cn*

**Abstract:** In this paper, we study a stochastic recursive optimal control problem in which the cost functional is described by the solution of a backward stochastic differential equation driven by G-Brownian motion. Under standard assumptions, we establish the dynamic programming principle and the related fully nonlinear HJB equation in the framework of G-expectation. Finally, we show that the value function is the viscosity solution of the obtained HJB equation.

(Joint work with Mingshang Hu.)

## **On multi-step MLE-processes in the problem of estimation of the solution of BSDE**

Yury Kutoyants

University of Maine, Le Mans, France

*kutoyants@univ-lemans.fr*

**Abstract:** We consider the problem of approximation of the solution of the BSDE in Markovian case, when the forward equation depends on some unknown finite-dimensional parameter. This approximation is done in two steps. First we estimate the unknown parameter and then we use this estimator and the solution of the corresponding PDE to provide the approximation of the

solution of the BSDE. We consider three models of the observations of the forward equations: dynamical systems with small noise (parameter in trend coefficient), ergodic diffusion process (parameter in trend coefficient) and discrete-time observations of diffusion process (parameter in volatility coefficient). It is shown that in all three cases the proposed approximations are asymptotically efficient. As estimators of the parameters we propose recently introduced multi-step MLE-processes. These estimator-processes provide *on-line* estimators, which can be easily calculated and have limit covariances coinciding with the limit covariance of the asymptotically efficient MLE.

(This is a review of joint works with L. Zhou, A. Abakirova, S. Gasparyan and of some new results.)

## On weak convergence of stochastic processes to stochastic integrals

Zhengyan Lin

Department of Mathematics, Zhejiang University, China

*zlin@zju.edu.cn*

**Abstract:** Weak convergence of various general functionals of partial sums of dependent random variables (statistics) to stochastic integrals now plays an important role in the modern probability theory and statistical theory. In this talk, we introduce the weak convergence of various general functionals of partial sums of causal processes to stochastic integrals driven by both the Brownian motion and Lévy  $\alpha$ -stable process.

## Observability estimate of backward stochastic schrödinger equations

Qi Lv

Sichuan University, China

*luqi59@163.com*

**Abstract:** In this talk, we introduce some recent results on the observability estimate for backward stochastic Schrödinger equations, which is motivated by the controllability problems of the

stochastic Schrödinger equations. As applications of this estimate, we not only solve the original controllability problem, but also an inverse problem for the backward stochastic Schrödinger equations.

## **Conditional mean-field SDEs and related stochastic control problems**

Jin Ma

University of Southern California, USA

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**Abstract:** We study a class of conditional mean-field SDEs (CMFSDEs, for short), in which the reference dynamics also involves the conditional law of the solution, hence “closed-loop” in nature. We investigate the well-posedness of the CMFSDEs in two particular settings, along with a non-Markovian stochastic control problems with partial observations and a problem regarding the “strategic insider trading equilibrium models”. In the former we consider a McKean-Vlasov stochastic control problem, with added aspects of path-dependence and partial observation. We prove Pontryagin’s Stochastic Maximum Principle and study the associated mean-field type Backward SDEs. In the latter we provide a rigorous theoretical basis for a general dynamic Kyle-Back type strategic insider trading equilibrium model in which the insider is allowed to have dynamic information of an asset rather than only the “long-lived” static ones; and tie the loose ends of some heuristic arguments in the literature of this problem.

(This talk is based on the joint works with Rainer Buckdahn, Juan Li, Yonghui Zhou, and Rentao Sun.)

## **BSDEs with diffusion constraint and viscous Hamilton-Jacobi equations with unbounded data**

Huyên Pham

Paris Diderot University, France

*pham@math.univ-paris-diderot.fr*



**Abstract:** We provide a stochastic representation for a general class of viscous Hamilton-Jacobi (HJ) equations, which has convexity and superlinear nonlinearity in its gradient term, via a type of backward stochastic differential equation (BSDE) with constraint in the martingale part. We compare our result with the classical representation in terms of (super)quadratic BSDE, and show in particular that existence of a solution to the viscous HJ equation can be obtained under more general growth assumptions on the coefficients, including both unbounded diffusion coefficient and terminal data.

(Joint work with Andrea Cosso (Paris Diderot) and Hao Xing (LSE).)

## **Multi-dimensional backward stochastic differential equations of diagonally quadratic generators**

Shanjian Tang

Department of Finance and Control Sciences, School of Mathematical Sciences,  
Fudan University, China  
*sjtang@fudan.edu.cn*

**Abstract:** We are concerned with adapted solution of a multi-dimensional BSDE with a “diagonally” quadratic generator, the quadratic part of whose  $i$ th component only depends on the  $i$ th row of the second unknown variable. Local and global solutions are given. Both John-Nirenberg and reverse Hölder inequalities for BMO martingales play a key role in our arguments.

(Joint work with Ying Hu at University of Rennes 1, France.)

## **On the law and sample paths of the solution to the heat equation with fractional noise**

Ciprian Tudor

Université Lille 1, France  
*Ciprian.Tudor@math.univ-lille1.fr*

**Abstract:** The solution to the heat equation with linear fractional noise is an interesting example of self-similar Gaussian process. We will discuss various properties of this process: the law,

the relation with the bifractional Brownian motion and the sample paths properties.

## **Partially observable optimal control problem of forward-backward stochastic control systems**

Zhen Wu

School of Mathematics, Shandong University, China

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**Abstract:** This talk is concerned with a partial observable optimal control problem derived by forward-backward stochastic control systems with correlated noises between the system and the observation. By virtue of a direct method, an approximate method, and a decomposition method, some optimality conditions and feedback optimal controls are derived. Closed-form optimal solutions are obtained in some detailed cases. A recursive utility problem is explicitly solved by the theoretical results obtained.

(Joint work with Guangchen Wang and Jie Xiong [SICON 2013, IEEE TAC 2015].)

## **Large deviation and moderate deviation principle for some measure-valued processes**

Jie Xiong

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**Abstract:** In this talk, I will show how to represent super-Brownian motion and Fleming-Viot process as the solution to a stochastic partial differential equation (SPDE) with non-Lipschitz coefficient. The uniqueness of the solution is obtained by relating the SPDE to a backward stochastic differential equation. The large and moderate deviation principles are obtained by some general results for SPDEs.

(This talk is based on some joint research with Fatheddin.)

## Rank-dependent utility maximization with bounded risk exposure

Zuoquan Xu

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**Abstract:** As risk measure based investment is now emerging as the industry standard by choices and by regulation, in this paper we analyze the optimal investment policies of rank-dependent utility maximizing investors who must manage risk exposure using a general law-invariant risk measure such as Value-at-Risk and Average Value-at-Risk. We overcome the difficulties arising from the discontinuity of the probability weighting function and obtain the closed-form optimal solution via the quantile formulation, change of variable technique and relaxation method.

(Joint work with Peizhen Ding.)

## Comparison theorems for backward stochastic Volterra integral equations and dynamic risk measures

Jiongmin Yong

Department of Mathematics, University of Central Florida, USA

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**Abstract:** To measure the risk for a portfolio process (not just a random variable, e.g., the payoff of certain European type contingent claim at the maturity), one could use the adapted solution to a backward stochastic Volterra integral equation (BSVIE, for short), provided some comparison theorems can be established for the BSVIEs. In this talk, some comparison theorems for BSVIEs will be presented. We will see that the results are very different from those for BSDEs.

(Joint work with Tianxiao Wang.)

## Some thoughts about pathwise stochastic analysis

Jianfeng Zhang

Department of Mathematics, University of Southern California, USA

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**Abstract:** Two fundamental subjects in stochastic calculus are stochastic integration and conditional expectation, which can be viewed as linear forward SDEs and backward SDEs, respectively. We shall view such equations as Path Dependent PDEs (PPDEs for short), and motivated by applications, we are interested in pathwise analysis of fully nonlinear PPDEs. It turns out that Dupires functional Ito calculus is a convenient tool for backward PPDEs, while for forward PPDEs the rough path analysis is appropriate. In this talk we shall introduce the pathwise Ito calculus, which unifies the functional Ito calculus and the rough path analysis, and thus provides a unified language for both forward PPDEs and backward PPDEs. As an application, we will also discuss viscosity solution of PPDEs.

## **First and second order necessary conditions for stochastic optimal controls**

Xu Zhang

School of Mathematics, Sichuan University, China

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**Abstract:** The purpose of this work is to establish the first and second order necessary conditions for optimal controls, using some techniques from classical variational analysis. The control system is governed by a stochastic differential equation, both drift and diffusion terms may contain the control variable and the control region is allowed to be nonconvex. Only one adjoint equation is introduced to derive the first order necessary condition; while for the second order necessary condition, only two adjoint equations are introduced.

(Joint work with H el ene Frankowska and Haisen Zhang.)

## **Random periodic processes, periodic measures and ergodicity**

Huaizhong Zhao

Loughborough University, UK

**Abstract:** We prove an ergodic theorem and a mean ergodic theorem in the random periodic regime on a Polish space. We introduce the idea of Poincaré sections and under the irreducible assumption on Poincaré sections, we obtain the weak convergence of the transition probabilities at the discrete time of integral multiple of the period. Thus we obtain the ergodicity of the invariant measure, which is the mean of the periodic measure over a period interval. The ergodicity and Poincaré sections give a clear geometric picture of random periodic processes. We establish the “equivalence” of random periodic processes and periodic measures. We prove that a random periodic path of a random dynamical system gives a periodic measure. Conversely, in general a periodic measure cannot give a random periodic path on the original probability space. But we can construct an enlarged probability space, on which the extended random dynamical system has a random periodic process. The law of the random periodic process is the periodic measure in the Markovian case. We further prove the strong law of large numbers (SLLN). This is a new class of random processes satisfying the ergodic theory and SLLN complimentary to the existing ergodic theory in the stationary regime.

(Joint work with Chunrong Feng.)

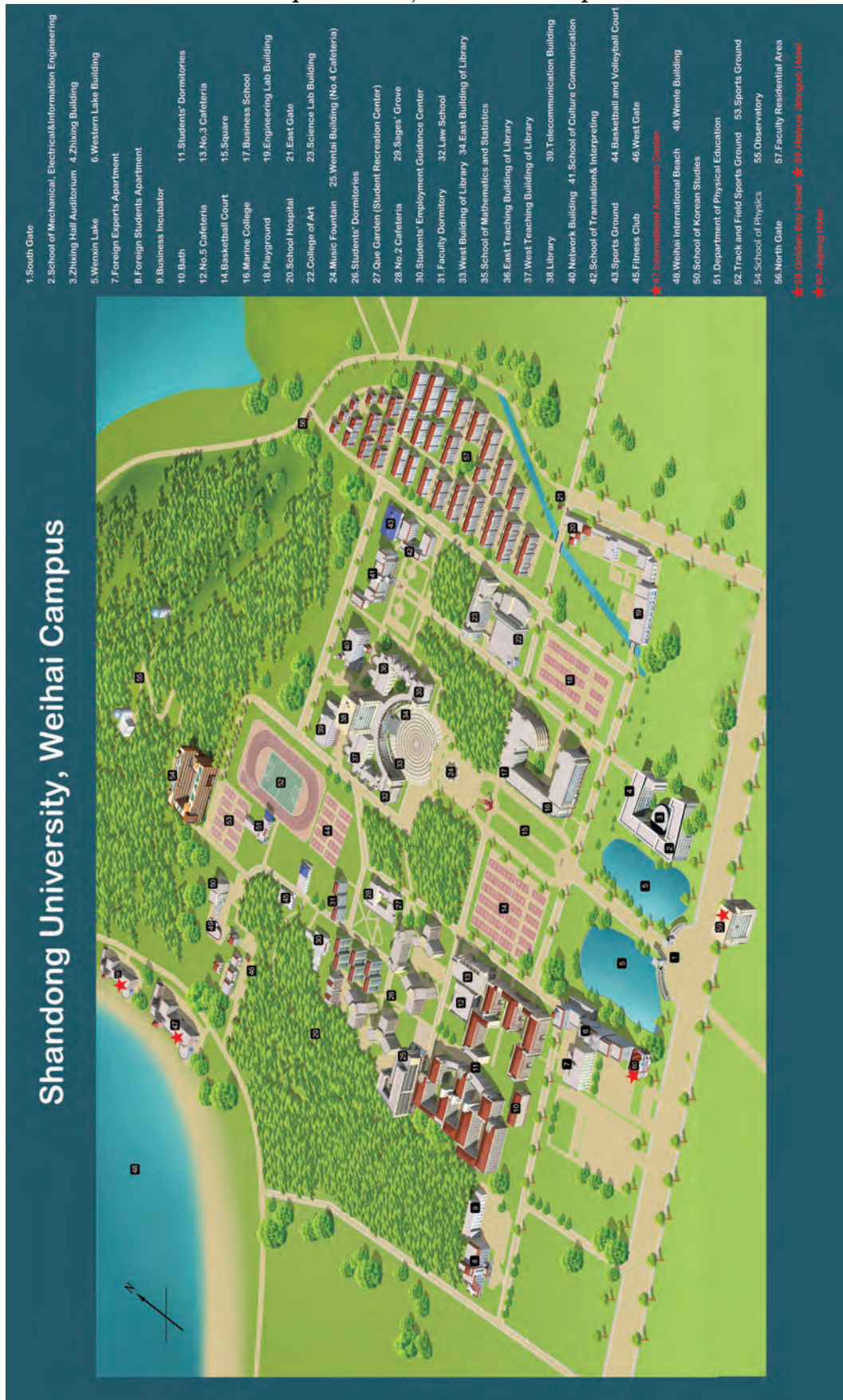
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# Appendix

## I. Map of SDU, Weihai Campus





# 山东大学(威海)校园示意图



- |                |                  |
|----------------|------------------|
| 1.南门           | 30.大学生就业创业指导中心   |
| 2.机电与信息工程学院    | 31.教工宿舍          |
| 3.知行楼报告厅       | 32.法学院           |
| 4.知行楼          | 33.圆西环路          |
| 5.文心湖          | 34.圆东环路          |
| 6.湖西楼          | 35.数学与统计学院       |
| 7.文轩苑(外籍专家公寓)  | 36.圆东教学楼         |
| 8.文华苑          | 37.圆西教学楼         |
| 9.大学生创业孵化园     | 38.图书馆           |
| 10.浴池          | 39.电信楼           |
| 11.文汇苑学生公寓楼群   | 40.网络楼           |
| 12.第五食堂        | 41.文学楼(文化传播学院)   |
| 13.文馨苑(第三食堂)   | 42.水化楼(翻译学院、马列部) |
| 14.篮球场         | 43.风雨操场          |
| 15.广场          | 44.蓝排球场          |
| 16.海洋学院        | 45.体育健身俱乐部       |
| 17.商学院         | 46.西门            |
| 18.塑胶运动场       | ★47.国际学术中心       |
| 19.工程实验楼       | 48.国际海水浴场        |
| 20.校医院         | 49.文萃苑           |
| 21.东门          | 50.韩国学院          |
| 22.艺术学院        | 51.体育教学部         |
| 23.科学实验楼       | 52.田径运动场         |
| 24.音乐喷泉        | 53.风雨操场          |
| 25.文泰苑         | 54.空间科学与物理学院     |
| 26.学生公寓楼群      | 55.天文台           |
| 27.葡园(大学生活动中心) | 56.北门            |
| 28.第二食堂        | 57.教工住宅二区        |
| 29.先贤林         | ★58.金海湾国际饭店      |
|                | ★59.海悦建国饭店       |
|                | ★60.锦江之星         |

## II. Hotel Information

The workshop is organized by the School of Mathematics of Shandong University, Jinan and Weihai. The workshop will take place in Weihai Campus of Shandong University. The hotel information is as follows:

1. International Academic Center, Shandong University, Weihai, the address is No. 130, North Huanhai Road, Gaoxin District, Weihai City, Shandong Province (威海国际学术交流中心-山东省威海市高新区北环海路130号), No. 47 in the map.
2. Golden Bay Hotel, the address is No. 128, North Huanhai Road, Gaoxin District, Weihai City, Shandong Province (威海金海湾国际饭店-山东省威海市高新区北环海路128号), No. 58 in the map.
3. Haiyue Jianguo Hotel, the address is No. 177, Wenhua West Road, Gaoxin District, Weihai City, Shandong Province (海悦建国饭店-山东省威海市高新区文化西路177号), No. 59 in the map.
4. Jinjiang Inn, the address is No. 182, Wenhua West Road, Gaoxin District, Weihai City, Shandong Province (锦江之星-山东省威海市高新区文化西路182号), No. 60 in the map.